Technical Notes

Prediction of Instability Zones in Liquid Rocket Engines

M. R. Beltran* and N. A. Frankel†
North American Aviation, Inc., Downey, Calif.

Nomenclature

m = burning-rate of propellant, fraction/in.
ΔP = pulse pressure, $(P_{max}-P_{min})$ lb/in.^{2}
r = radial coordinate, in.
r_{an} = radius of annulus, in.
ΔV = absolute value of the relative velocity between gases and drops, in./sec
z = axial coordinate, in.
$\mathfrak{A}$ = combustor contraction ratio A_c/A_t, dimensionless
$\mathfrak{L}$ = burning-rate parameter $(r_{an}m/\mathfrak{A})$, dimensionless
θ = angular coordinate, dimensionless

I. Introduction

COMBUSTION instability zones in a liquid rocket engine are determined in this note by combining a nonlinear instability model[1] with a propellant vaporization program,[2] providing an analytical framework for determining the relationship of design parameters to stability. The analysis determines the zones of an engine in which a tangential mode of high-frequency instability is most easily initiated.

II. Method of Analysis

The Priem-Guentert instability model[1] considers the nonlinear conservation equations with mass addition using a steady-state vaporization expression for the burning rate. From such a model, important nonlinear phenomena are predicted, i.e., 1) stability dependence on disturbance amplitude, 2) the limiting amplitude of pressure oscillations, and 3) nonsinusoidal waveforms. This model applies to a one-dimensional annulus of small length (Δz) and thickness (Δr) shown in Fig. 1.

Applying the results of this model, a rocket engine can be analyzed by incrementally dividing the combustion chamber into annular nodes in the r and z directions. Steady-state properties at each annular node or position in the chamber are computed from the Priem-Heidmann propellant vaporization program.[2] These steady-state propellant properties and the curves from the instability model[1] are used to determine the stability of the node. This process is repeated for each node to determine a stability map of the entire engine.

For a vaporization-controlled combustion process, the significant parameters affecting stability[1] are $\mathfrak{L}$ and ΔV. If one-dimensional flow is assumed, m, $\mathfrak{A}$, and ΔV are functions of z only; $\mathfrak{L}$ is a function of r and z. The contraction ratio of any annulus at z is equal to the contraction ratio of the chamber at z.

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* Research Engineer, Propulsion Analysis, Space and Information Systems Division; now Propulsion Specialist, Dynamic Science Corporation, South Pasadena, Calif. Member AIAA.

† Research Engineer, Propulsion Analysis, Space and Information Systems Division; now Ph.D. Candidate, Department of Chemical Engineering, Stanford University, Stanford, Calif.

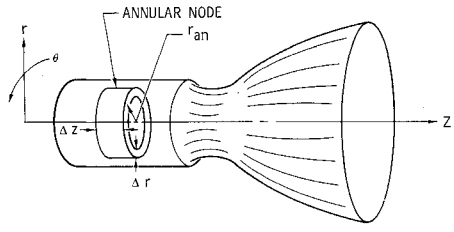

Fig. 1 Rocket engine model.

To illustrate this nodal method, combustion of a heptane spray in gaseous oxygen is considered. This method can be used on other propellant combinations as well. Calculations are based on a polydispersed spray with an initial mass-median drop radius of 75 μ. A logarithmiconormal distribution function[3] with a standard deviation of 2.3 was used to describe the spray. The other parameters are: chamber diameter, 18 in.; chamber pressure, 300 psia; chamber contraction ratio, 3.15; initial drop velocity, 1200 in./sec.; and initial drop temperature, 500°R.

Results

Using the outlined chamber parameters, ΔV_z and m are computed as functions of z and plotted in Fig. 2. Using the vaporization program,[2] only the relative velocity in the z direction is computed; however, in the injection zone there are large gradients and recirculation zones due to the viscous action between the sprays and combustion gases, so that velocity differences exist in the r and θ directions as well. For the engine being considered, a level of turbulence equivalent to 420 in./sec is reasonable in view of the measurements made by Hersch.[4] If it is assumed that the drops are unaffected by turbulent oscillations, which is justified with the drop size group considered, this velocity can be vectorially added to ΔV_z thereby obtaining the total ΔV. A mean drop size was selected to represent the average ΔV, since the smallest droplets will follow the gas velocity and the largest droplets will lag. The choice of a mean drop size serves to illustrate the method used; however, this does not imply necessarily that a mean drop is a good representation of the spray drop distribution. The method is quite flexible in that

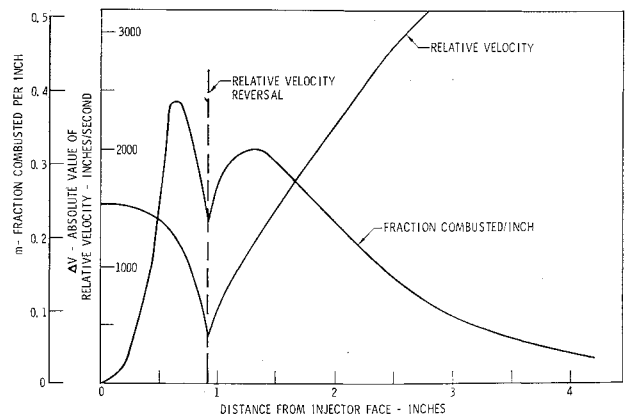

Fig. 2 Computed profiles of steady-state chamber parameters.

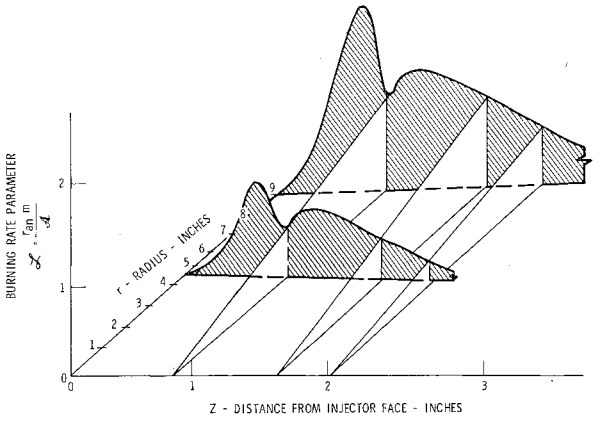

Fig. 3 Computed burning rate parameter distribution in combustion chamber.

it can be readily extended to treat distributed relative velocities.

In Fig. 3, $\mathfrak{L}$ is plotted vs r and z. Since $\pounds$ is directly proportional to r ($\pounds = r_{an}m/\mathfrak{L}$), it approaches zero at the centerline of the engine. Figure 4 is a plot of the pressure amplitude ($\Delta P/P_c$) required to sustain a wave[1] for various values of the parameter ΔV. The upper boundary represents the equilibrium pressure amplitude that the wave will approach at steady-state, and the lower boundary represents the minimum pressure amplitude necessary to initiate instability. Only results for the lower boundary are employed since, in a full grown instability droplet, shattering probably plays an important role.

Figures 2–4 were cross plotted to determine pulse pressure contour lines in the r-z plane as shown in Fig. 5. These isobaric lines describe the tangential threshold pulse required to initiate instability. It can be seen that the sensitive zone is from 0.2 to 2.7 in. from the injector face. For the case considered, this zone is largest at the outer radius. In general, the largest zone will occur where $\pounds$ and ΔV determine a point on the locus of minima of Fig. 4. Thus, for a larger diameter engine, the maximum sensitive zone may occur at some intermediate radius. Figure 5 shows that there is an extremely sensitive region occurring when the relative velocity between droplets and gas reverses direction, i.e., several inches from the injector face. This result is consistent with experimental observations of Peoples and Baker,[5] who found that there is a sensitive region in liquid rocket engines near the injector face.

Application

The method presented can be very useful in the design of stable rocket engines. For an unbaffled engine, changes in injector parameters and propellant properties will have an opposite effect on minimum sensitivity depending upon which side of the locus of minima curve (Fig. 4) the node falls. Hence, the same design changes may have the opposite effect on stability for two different engine configurations. Since it is difficult to change one of the parameters independently of

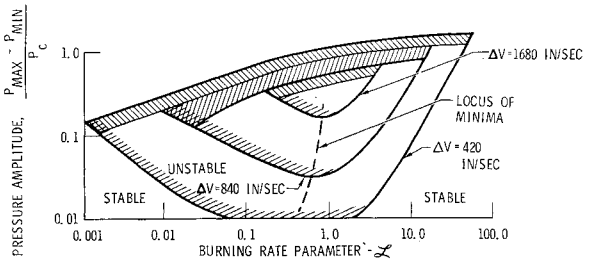

Fig. 4 Stability limit curves (see Ref. 1).

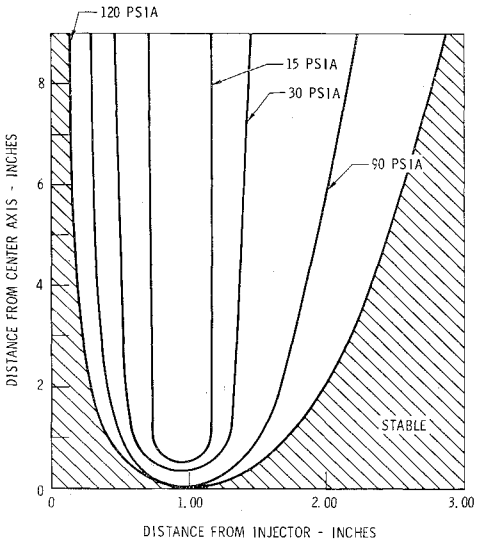

Fig. 5 Pulse pressure contour lines defining unstable zones.

the others, it is recommended that any proposed design change be analyzed in detail by the methods presented here before any conclusions are drawn.

Figure 5 also presents important information for designing a stable baffled injector. For the case considered, the results indicate that the effective baffle configuration should extend at least 2.7 in. from the injector face at the outer periphery and 1.5 in. at the center to protect the sensitive zone. However, since the model used here[1] is one-dimensional, an added length of about 1 in. may be reasonable to allow for spillover (a three-dimensional effect not considered here).

III. Conclusion

Although the results presented here are reasonable in light of liquid rocket engine testing experience, it must be kept in mind that the basis of the model is a one-dimensional description of the transient processes. Two- and three-dimensional solutions would provide a description of the multidimensional nonlinear wave interactions and permit a more realistic consideration of the chamber boundary conditions which may significantly affect the results.[6–8] However, it is felt that the agreement noted between the one-dimensional model and experimental findings indicates that the one-dimensional approximation leads to meaningful results. Thus, the model is valuable from an engineering standpoint since it relates design parameters directly to stability determination.

Results of this study show that: 1) the amplitude and position of a pressure disturbance required to initiate instability can be determined, thereby defining a sensitive zone; 2) this sensitive zone extends several inches from the injector face; 3) the most sensitive region in the zone occurs where the average droplets are moving the slowest relative to the gases; and 4) approximate baffle length can be determined, i.e., baffles must be extended past the instability zone.

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Dynamic Loading of Structural Models by Electrostatic Forces

J. P. O'Neill*
TRW Space Technology Laboratories, Redondo Beach, Calif.

Nomenclature

E = electric intensity of field
ϵ = absolute permittivity of medium
σ = charge per unit area
δ = 1 for rationalized system of units, = 4π for unrationalized system
f = forcer per unit area
C = capacitance between two conductors
Q = charge
V = potential difference between two conductors
A = area of each conductor forming capacitance C
t = separation distance between conductors forming capacitance C
F = total force
ϵ_0 = absolute permittivity of free space, i.e., a vacuum
ϵ_r = relative permittivity or dielectric constant of medium
d = separation between plates, in.
p = pressure, psi
V_p = potential difference between plates, v.

Introduction

RECENT investigations[1–3] of the buckling of shells under dynamic loading have required the development of experimental techniques for exciting various structural modes. The diamond-shaped buckling modes of an 8-in.-diam mylar cylinder were excited with an air-pressure step with a 3-msec rise time; but the desired excitation of the cylinder's 3000 cps ring mode could not be obtained with air loading. By direct application of electrostatic forces, however, the thin-walled cylinder was buckled and the loading rate was adequately high to excite the ring mode. Forcing rates greatly in excess of any structural response rates can be obtained with electrostatic loading systems. The principles of such loading, derived from the basic phenomena of the interaction of electric charges and electric fields, may be uniquely applicable in many investigations and processes.

Interaction of Electric Charges and Electric Fields

The potentials and geometric arrangement of a system of conductors separated by insulating free space or dielectrics will determine the equilibrium distribution of charge and field and therefore the distribution of forces on all surfaces. By making some of the conductors and solid dielectrics serve as the model structure, a system is produced in which the loading can be changed in the very short time required to charge a small capacitor.

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* Manager, Experimental Mechanics, Engineering Mechanics Laboratory.

By Coulomb's theorem, the electric intensity E of the field near the surface of a conductor carrying a charge σ per unit area is [4, 5]

$$E = \delta\sigma/\epsilon \tag{1}$$

where ϵ is the absolute permittivity of the medium outside the conductor, and σ is introduced in order that its value, either 1 or 4π, will allow the use of either a rationalized or unrationalized system of units. Unless otherwise noted, all equations are dimensionally homogeneous and applicable for any consistent system of units.

The most basic parameter of interest is the electrostatic force on a conductor. The force per unit area at the surface of a conductor is[4,5]

$$f = \delta\sigma^2/2\epsilon \tag{2}$$

or, through the relationship expressed by Coulomb's theorem, it is

$$f = \epsilon E^2/2\delta \tag{3}$$

Regardless of whether σ is a positive or negative charge and regardless of the resulting direction of the vector field E, the direction of the force is always outwards from the surface, i.e., the surface of the conductor is pulled toward the dielectric; there is consequently never a push against the surface of a conductor. This suggests that the term "push-pull," sometimes applied to certain configurations of electrostatic transducers, is inappropriate (at least this terminology and the associated analyses have sometimes been misleading).

With Eqs. (2) and (3) considered as basic, it remains necessary to arrive at distributions of σ and E. Although fields of arbitrary geometric configuration can be plotted by the relaxation method, lumped-constant systems consisting of arrangements of capacitors are analytically determinate and adequate for the analysis of the loading systems usually desired.

The force per unit area on each inner surface of a parallel-plate capacitor is

$$f = \epsilon V^2/2\delta t^2 \tag{4}$$

where V is the potential difference between the two conductors and t is the thickness of the dielectric that separates the plates. Since the electrostatic force on a conductor is always outward from its surface, this force is manifest as an attraction between the plates.

The total force on each of the inner surfaces of a parallel-plate capacitor of area A is given by

$$F = \frac{\delta^2 A}{2\epsilon} = \frac{\epsilon E^2 A}{2\delta} = \frac{\epsilon V^2 A}{2\delta t^2} \tag{5}$$

when Eqs. (2–4) are multiplied by the area, and is given by

$$F = CV^2/2t = QV/2t \tag{6}$$

when the total force is desired in terms of capacitance C between the two conductors or charge Q carried by either conductor, of plate separation t, and of potential difference V.

Effect of the Dielectric

In the foregoing equations, the absolute permittivity ϵ of the dielectric is given by

$$\epsilon = \epsilon_0\epsilon_r \tag{7}$$

where ϵ_0 is the absolute permittivity of free space, i.e., in a vacuum and ϵ_r is the relative permittivity or dielectric constant of the medium separating the conductors. In an electrostatic loading system for structural models, at least part of the space between the conductors would be occupied by either a vacuum or a fluid dielectric in order to allow the structural response being investigated. If air is the fluid dielectric, the resulting ϵ is about the same as ϵ_0, since ϵ_r for air